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Stationary regime of random resistor networks under biased percolation

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Abstract
The state of a two-dimensional random resistor network, resulting from the simultaneous evolutions of two competing biased percolations, is studied in a wide range of bias values. Monte Carlo simulations show that when the external current I is below the threshold value for electrical breakdown, the network reaches a steady state with nonlinear current–voltage characteristics. The properties of this nonlinear regime are investigated as a function of different model parameters. A scaling relation is found between $\langle R\rangle/\langle R\rangle_0$ and I/I_0, where $\langle R\rangle$ is the average resistance, $\langle R\rangle_0$ the linear regime resistance and I_0 the threshold value for the onset of nonlinearity. The scaling exponent is found to be independent of the model parameters. A similar scaling behaviour is also found for the relative variance of resistance fluctuations. These results compare well with resistance measurements in composite materials performed in the Joule regime up to breakdown.

1. Introduction and model

Electrical breakdown of disordered media has been widely studied over the last 20 years [1–10]. This is due to its relevant implications on material technology and on fundamental aspects related to the understanding of the response properties of disordered systems to high external stresses [1, 2]. It is well known that the application of a finite stress (electrical or mechanical) to a disordered material generally implies a nonlinear response, which leads to a catastrophic behaviour in the high stress limit [1, 2]. Percolation theory provides a powerful approach for studying breakdown phenomena of disordered media [11, 12]. In this framework, several models have been proposed to describe the electrical breakdown of granular metals and of

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conductor–insulator composites in terms of critical phenomena near the percolation threshold [4–7]. Furthermore, the critical exponents characterizing these behaviours have been both theoretically calculated and measured in several materials [1, 2, 4–7, 10]. Nevertheless, few attempts have been made so far to describe the behaviour of disordered media over the full range of applied stress [7, 9]. The understanding of breakdown phenomena in the full dynamical regime is thus unsatisfactory at the present time. On the other hand, important information is expected from such a study, such as precursor effects, the role of disorder, the predictability of breakdown, etc [7, 9].

The aim of this paper is to present a percolative model of sufficient generality to address the above issues. To this purpose, we consider a random resistor network (RRN) [11] in which two competing percolations are present, defect generation and defect recovery, which determine the values of the elementary network resistances. Both processes are driven by an external current and by the heat exchange between the network and the thermal bath. Monte Carlo simulations are performed to explore the network evolution in a wide range of bias values. A stationary state or an irreversible breakdown of the RRN can be reached depending on the value of the applied current. By focusing on the steady state, we study the resistance and the resistance noise properties. We found that the average network resistance and the relative resistance noise scale with the ratio of the applied current to the current value corresponding to the onset of nonlinearity. These results are discussed in connection with measurements in composite materials and in conducting polymers [9, 13].

We consider a two-dimensional, square-lattice RRN of total resistance R, made of $N_{\rm tot}$ resistors with resistance r_n. We take a square geometry $N \times N$, where N determines the linear size of the network. A constant current I is applied through electrical contacts realized by perfectly conducting bars on the left-hand and right-hand sides of the network. As a consequence, a current i_n is flowing in the nth resistor. The RRN interacts with a thermal bath at temperature T_0 and the resistances r_n are taken to depend linearly on the local temperature T_n, according to

$$r_n(T_n) = r_0[1 + \alpha(T_n - T_0)] \qquad (1)$$

where r_0 is the resistance value of the elementary resistor at the temperature T_0 and α is the temperature coefficient of the resistance. The local temperatures are calculated as in [10]:

$$T_n = T_0 + A\left[r_n i_n^2 + \frac{B}{N_{\rm neig}}\sum_{l=1}^{N_{\rm neig}}\left(r_l i_l^2 - r_n i_n^2\right)\right]. \qquad (2)$$

In this expression, $N_{\rm neig}$ is the number of first neighbours of the nth resistor, the parameter A, measured in (K/W), describes the heat coupling of each resistor with the thermal bath and it determines the importance of Joule heating effects. The parameter B is taken to be equal to 3/4 to provide a uniform heating in the perfect network configuration. We notice that equation (2) implies an instantaneous thermalization of each resistor at the value T_n, and then, by adopting equation (2), we are neglecting for simplicity time-dependent effects which are discussed in [5].

In the initial state of the network, $I = 0$, $T_n \equiv T_0$ and all the resistors are identical $r_n \equiv r_0$. The evolution of the RRN arises from the presence of two competing percolations. The first consists of generating fully insulating defects (broken resistors). This process occurs with probability

$$W_{\rm D} = \exp[-E_{\rm D}/k_{\rm B}T_n] \qquad (3)$$

where $E_{\rm D}$ is a characteristic activation energy and $k_{\rm B}$ the Boltzmann constant [10]. The second percolation consists of recovering the insulating defects. This process occurs with a

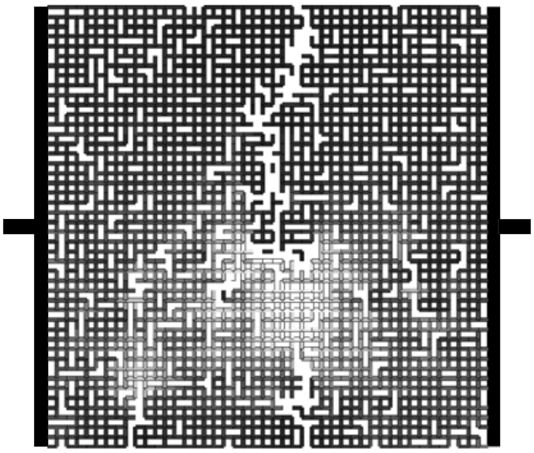

Figure 1. Network evolution near the electrical breakdown. Different grey levels from black to white correspond to increasing values of r_n from 1 to 3 (Ω).

probability W_R expressed as in equation (3) but with a different activation energy E_R. As a result of the competition between these two percolations, depending on the parameters which specify the physical properties of the system and depending on the external conditions (bias current and bath temperature), the RRN can reach a steady state or the percolation threshold. In the first case, R fluctuates around its average value $\langle R \rangle$, while in the second case, an irreversible breakdown occurs, i.e. R diverges due to the existence of at least one continuous path of defects between the upper and lower sides of the network [11]. By focusing on the effect of the external current, we define I_b as the greatest current value for which the RNN is stationary. We notice that for biased percolation the following expression, $E_R < E_D + k_B T \ln[1 + \exp(-E_D/k_B T_0)]$, represents a necessary condition for the existence of a steady state [14]. Monte Carlo simulations are performed according to the following procedure:

(i) starting from the perfect lattice with given local currents and temperatures, i_n and T_n,

(ii) resistances r_n are changed according to equation (1) and defects are generated with probability W_D;

(iii) the currents i_n are calculated by solving Kirchhoff's loop equations; the local temperatures are updated;

(iv) the temperature dependence of the resistances r_n is again accounted for and defects are recovered with probability W_R;

(v) R, i_n and T_n are finally calculated and this procedure is iterated from (ii) until electrical breakdown or steady state is achieved. In the last case the iteration runs long enough to allow a fluctuation analysis to be performed. Each iteration step can be associated with an elementary time step on an appropriate timescale (to be calibrated with experiments).

As reasonable values of the parameters, simulations have been performed by taking: $N = 75$ (except when differently specified), $T_0 = 300$ (K), $\alpha = 10^{-3}$ (K^{-1}), $A = 5 \times 10^5$ (K W^{-1}), $E_D = 0.17$ (eV). Several values of E_R and r_0 have been considered: $0.026 \leqslant E_R \leqslant 0.16$ (eV) and $1 \leqslant r_0 \leqslant 10$ (Ω). The values of the external current range between 0.001 and 3 (A).

2. Results

We show in figure 1 a picture of the RRN near the electrical breakdown. In this case we have taken $N = 45$, $r_0 = 1$ (Ω), $E_R = 0.10$ (eV) and $I = 0.5$ (A), i.e. $I > I_b = 0.45$ (A). The different levels of grey correspond to different values of r_n. We can clearly see that, with respect to the initial state (perfect network), the network has evolved to a disordered state associated with the formation and growth of a channel of broken resistors elongated in the direction perpendicular to the applied current. This kind of damage pattern reproduces well the experimental pattern observed in metallic lines failed as a consequence of electromigration [15]. Typical evolutions of R are shown in figure 2 at increasing bias values. In this case $N = 75$ while all the other parameters are the same of figure 1. The thinner curves refer to the steady state regime while the thicker curve refers to a RRN undergoing electrical breakdown ($I > I_b = 0.75$). In the steady state two regimes can be identified: an ohmic regime (lower two curves) and a nonlinear regime characterized by a significant increase of resistance (remaining curves). By focusing on the steady state regime, we report in figure 3 the average resistance $\langle R\rangle$ as a function of the applied current. The different curves correspond to different values of r_0, i.e. to RRNs of different initial resistance, while the recovery activation energy is $E_R = 0.026$ (eV). Each value has been calculated by considering the time average on a single realization and then making the ensemble average over 20 realizations. At low biases the average resistance takes a constant value $\langle R\rangle_0$ which represents the intrinsic linear response property of the network (ohmic regime). When I is above a threshold value I_0 (threshold for the nonlinearity onset), $\langle R\rangle$ increases with bias until the applied current reaches the I_b value, above which the RRN undergoes electrical breakdown. Thus, in the following, we indicate with $\langle R\rangle_b$ the average value of R at $I = I_b$, i.e. the last stable value of the resistance. Figure 3 also shows that by increasing r_0 and thus the initial network resistance, both I_0 and I_b decrease. Precisely, we have found $I_b \sim R_0^{-\delta}$ and $I_0 \sim R_0^{-\delta}$ with $\delta = 0.51 \pm 0.01$. Therefore the ratio $I_b/I_0 = 28 \pm 1$ is independent of the initial network resistance. Moreover, we have also found $\langle R\rangle_b/\langle R\rangle_0 = 1.85 \pm 0.08$.

The effect of the recovery activation energy on the steady state is shown in figure 4, which reports the ratio $\langle R\rangle/\langle R\rangle_0$ as a function of the applied current for different values of E_R (in this case all the curves are obtained for $r_0 = 1$ (Ω)). The overall behaviour is similar to that shown in figure 3: an ohmic regime at low bias is followed by a nonlinear regime for $I > I_0$. Moreover, by increasing E_R both I_0 and I_b decrease and the stability region is strongly reduced. Nevertheless, an important difference between the effect of varying the initial network resistance and that of varying E_R is that, in the last case, the ratio I_b/I_0, exhibits a significant dependence on E_R, as shown in figure 5. To investigate the existence of scaling relations and their universality, figure 6 reports the log–log plot of the relative variation of the average resistance, $(\langle R\rangle - \langle R\rangle_0)/\langle R\rangle_0$, as a function of the ratio I/I_0 for different values of r_0 and E_R. The plot shows that all these curves collapse onto a single one and that the relative variation of $\langle R\rangle$ as a function of I/I_0 exhibits a power law behaviour. We conclude that, the average resistance follows the scaling relation

$$\frac{\langle R\rangle}{\langle R\rangle_0} = g(I/I_0) \qquad g(I/I_0) \simeq 1 + (I/I_0)^\theta \tag{4}$$

with the scaling exponent $\theta = 2.1 \pm 0.1$ being independent of both the initial resistance of the RRN and the recovery activation energy. Other simulations, performed on rectangular networks, confirm the same value for θ. The quadratic dependence of $\langle R\rangle$ on I, found here, can be understood in the spirit of mean-field theory when we consider that $\Delta R \approx \alpha R_0 \Delta T$ and $\Delta T \propto A R_0 I^2$. Moreover, recalling the above reported results concerning the ratio I/I_0,

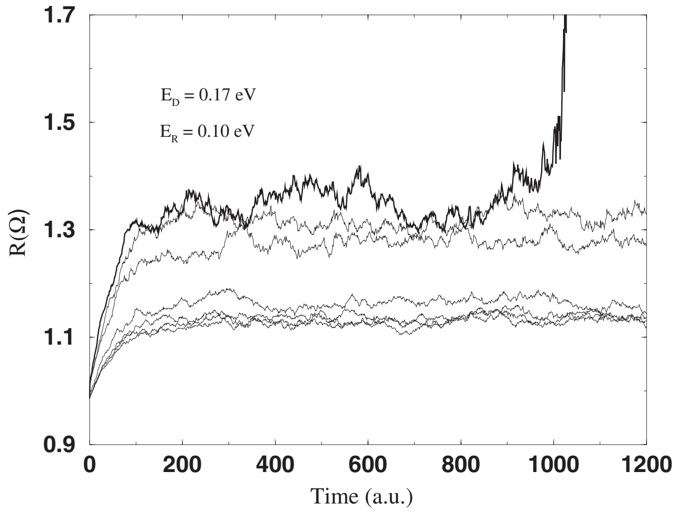

Figure 2. Resistance as a function of time at increasing bias values. The thinner curves correspond to the steady state regime and they are obtained, going from bottom to top, for $I = 0.01$, 0.05, 0.10, 0.35, 0.70, 0.75 (A). The thicker curve is obtained for $I = 0.78$ (A) and it corresponds to a RRN undergoing electrical breakdown.

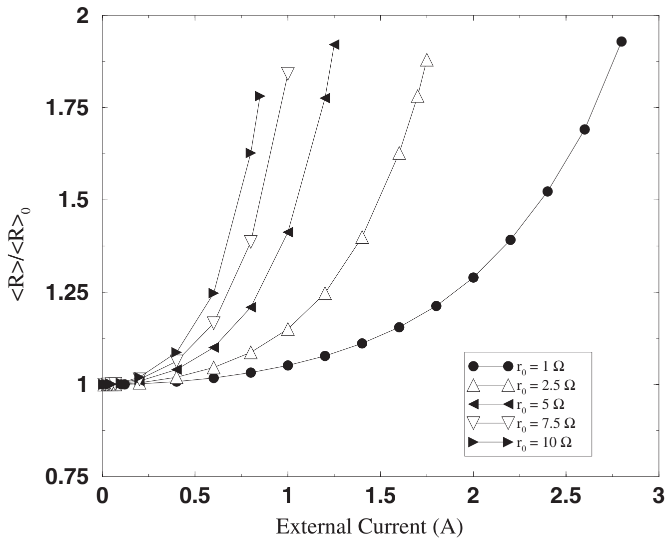

Figure 3. Normalized average resistance versus external bias. We take $E_{\mathrm{R}} = 0.026$ and $E_{\mathrm{D}} = 0.167$ (eV), while the value of r_0 ranges between 1 and 10 Ω.

equation (4) explains the independence of the ratio $\langle R\rangle_b/\langle R\rangle_0$ on the initial RRN resistance (figure 3) and, by contrast, its significant dependence on E_{R}, as shown in figure 4. All these results well agree with recent measurements in the Joule regime of carbon high-density polyethylene composites reported in [9].

The resistance fluctuations are then analysed for different values of E_{R} and r_0. Figure 7 reports the relative variance of resistance fluctuations, $\Sigma \equiv \langle \Delta R^2\rangle/\langle R\rangle^2$, as a function of the external current. Curves 1, 2 and 3 (with full circles) show Σ for $r_0 = 1$ (Ω) and

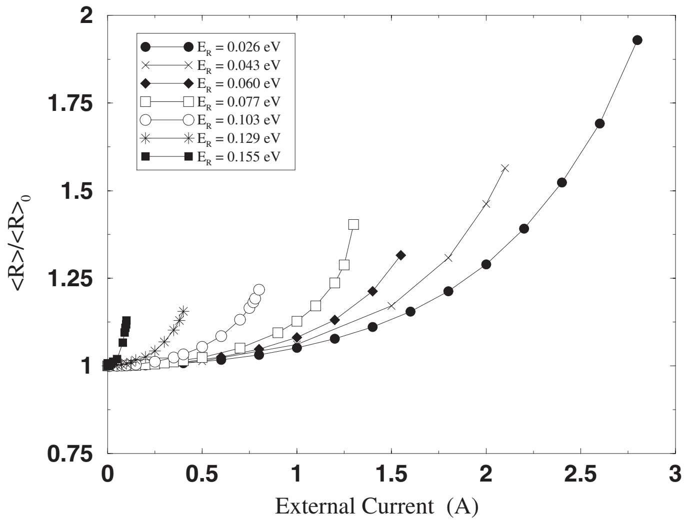

Figure 4. Normalized average resistance versus external bias. We take $r_0 = 1$ (Ω), $E_D = 0.167$ (eV) while the values of E_R range between 0.026 and 0.155 (eV).

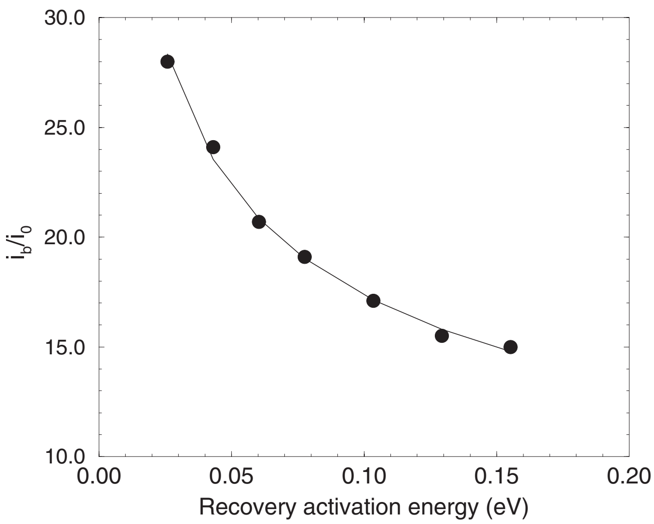

Figure 5. Plot of the ratio I_b/I_0 as function of the recovery activation energy. The curve is a fit with a power law $I_b/I_0 \sim E_R^{-0.36}$.

$E_R = 0.060$, 0.043, 0.026 (eV) respectively, while the curves belonging to set 3 are obtained for $E_R = 0.026$ (eV) and different values of r_0. Figure 7 points out the existence of two different noise regimes. The first regime occurs for $I < I_0$, i.e. when Joule heating effects are negligible. This noise arises from two random percolations and represents an intrinsic noise of the RRN, depending only on the values of E_D and E_R [14]. The second regime occurs when $I > I_0$ and the value of Σ is found to become strongly dependent on the external current. By plotting Σ/Σ_0 as a function of I/I_0 we have found that all the data of figure 7 collapse onto a single curve, as shown in figure 8. Moreover, a power law behaviour is observed in

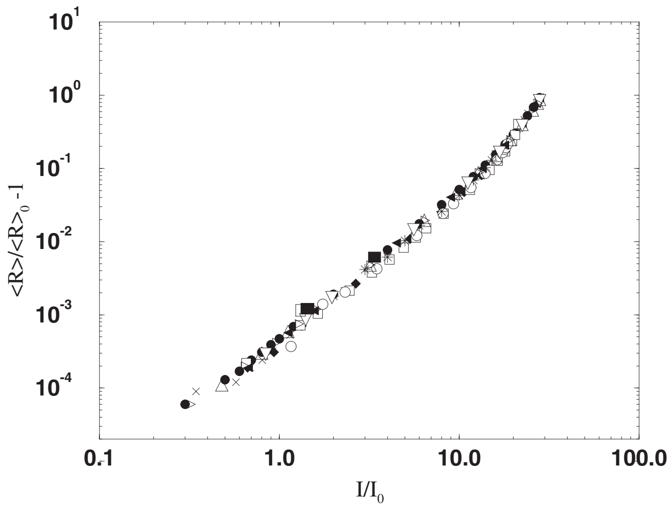

Figure 6. Log–log plot of the relative variation of resistance versus I/I_0. Data shown in this figure are the same of those reported in figures 3 and 4.

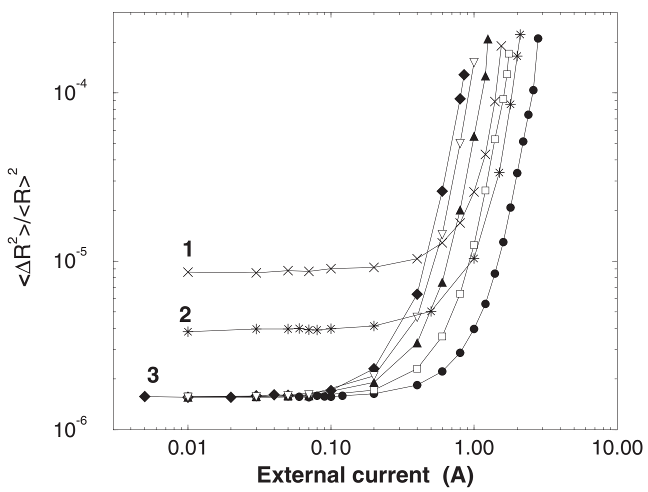

Figure 7. Relative variance of resistance fluctuations as a function of the external bias. Curves 1, 2, 3 refer to $r_0 = 1.0$ (Ω) and $E_R = 0.060$, 0.043, 0.026 eV, respectively. The five curves belonging to set 3 are obtained with $r_0 = 1.0$ (Ω) (full circles), 2.5 (Ω) (open squares), 5.0 (Ω) (full triangles), 7.5 (Ω) (open triangles), 10.0 (Ω) (full diamonds).

the pre-breakdown region. Therefore, we can conclude that in the pre-breakdown region the relative variance of resistance fluctuations follows the scaling relation

$$\frac{\Sigma}{\Sigma_0} = f(I/I_0) \qquad f(I/I_0) \simeq 1 + (I/I_0)^{\eta} \tag{5}$$

where the scaling exponent is $\eta = 4.1 \pm 0.1$. This value of η agrees with the values obtained from electrical noise measurements in conducting polymers [13].

In conclusion, we have studied by Monte Carlo simulations the stationary regime of RRNs resulting from the simultaneous evolutions of two competing percolations. The two percolations consist of generating (recovering) fully insulating defects which are driven by an

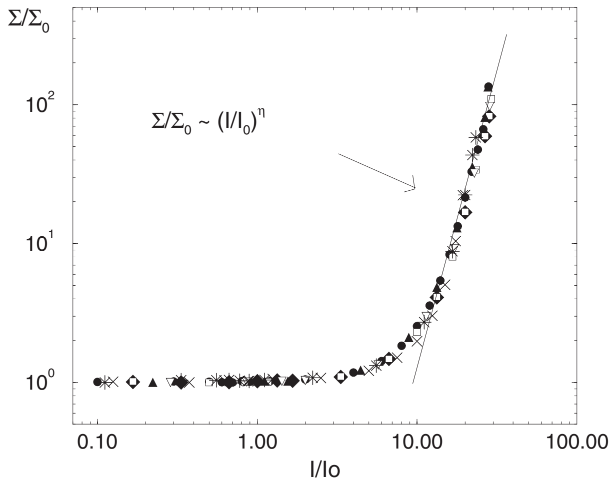

Figure 8. Log–log plot of the relative variance of resistance fluctuations normalized to the same quantity calculated in the linear regime versus I/I_0. A power law fit is shown in the pre-breakdown regime.

external current and by the heat exchange with a thermal bath. We have analysed the behaviour of the average resistance and of the relative variance of resistance fluctuations over a wide range of the applied current and as a function of different model parameters. We have found that both these quantities follow a scaling relation in terms of the ratio between the applied current and the current value corresponding to the nonlinearity onset. Both scaling exponents are found to be independent of the model parameters. These results compare well with resistance measurements in composite materials performed in the Joule regime up to breakdown [9] and with noise measurements in conducting polymers [13].

Acknowledgments

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